

# Sensitivity Analysis and Uncertainty Quantification for BDT

Addressing Gaps and Challenges to Successful BDT Implementation:

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$$\frac{\mathbf{p}}{m} \Delta t, \mathbf{p} + \mathbf{F} \Delta t, t + \Delta t) d^3 \mathbf{r} d^3 \mathbf{p} = f(\mathbf{r}, \mathbf{p}, t) d^3 \mathbf{r} d^3 \mathbf{p}$$

$$dN = f(\mathbf{r}, \mathbf{p}, t) d^3 \mathbf{r} d^3 \mathbf{p}$$

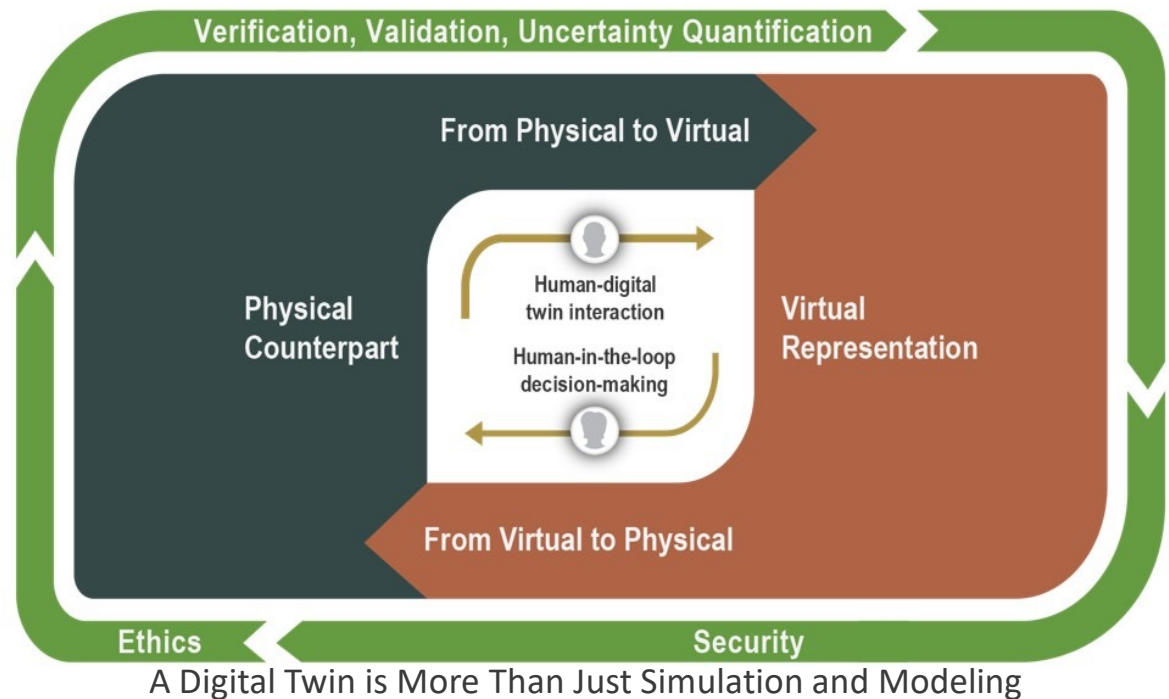
$$\frac{\mathbf{p}_i}{m_i} \cdot \nabla f_i + \mathbf{F} \cdot \frac{\partial f_i}{\partial \mathbf{p}_i} = \left( \frac{\partial f_i}{\partial t} \right)_{\text{coll}}$$

$$\int A F_j \frac{\partial f}{\partial p_j} d^3 \mathbf{p} = -n F_j \left\langle \frac{\partial}{\partial p} \right\rangle$$

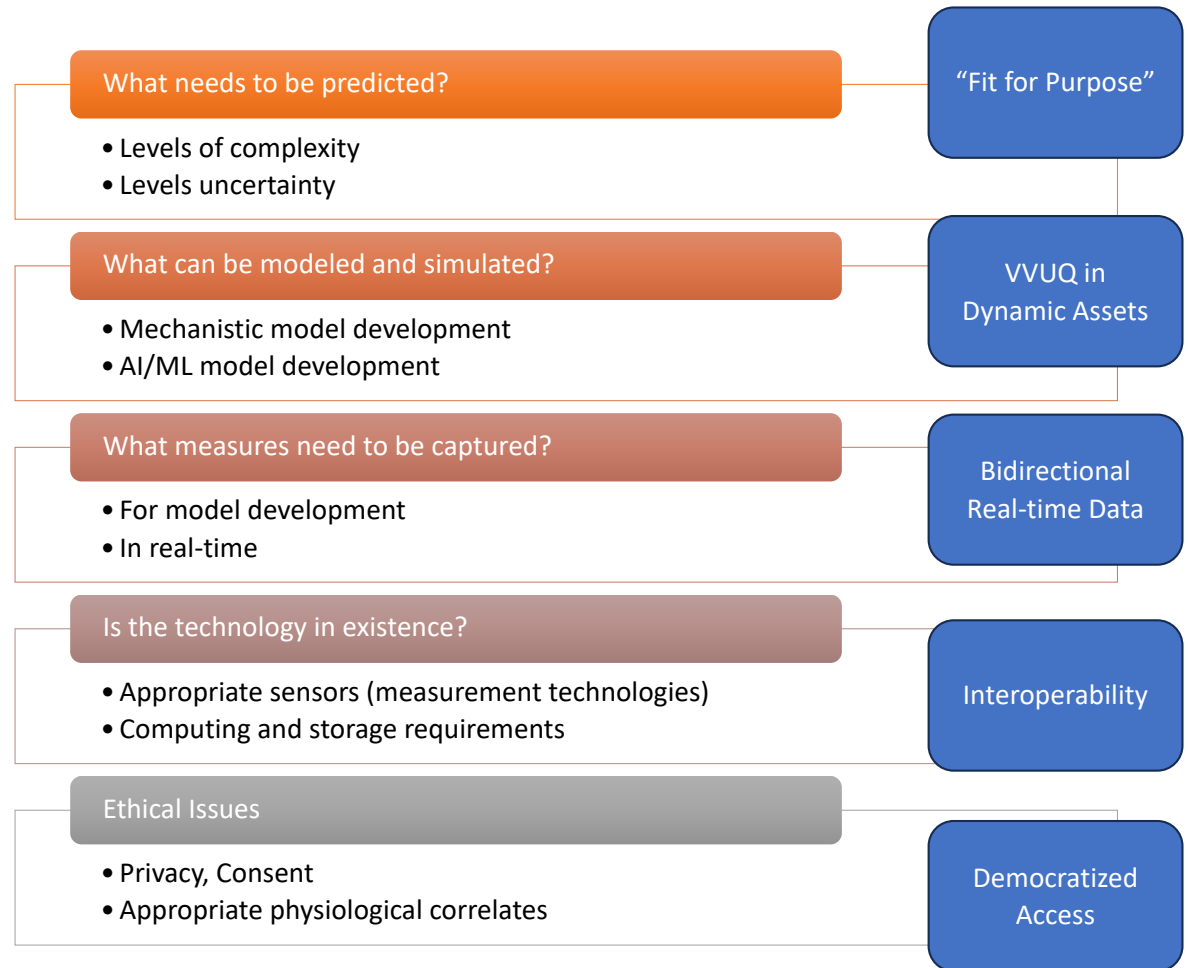
$$\frac{\partial}{\partial t} + \frac{\mathbf{p}}{m} \cdot \nabla + \mathbf{F} \cdot \frac{\partial}{\partial \mathbf{p}}$$

# NASEM Definition of a Digital Twin

"A digital twin is a set of virtual information constructs that mimics the structure, context, and behavior of a natural, engineered, or social system (or system-of-systems), is dynamically updated with data from its physical twin, has a predictive capability, and informs decisions that realize value. The bidirectional interaction between the virtual and the physical is central to the digital twin."



# Biomedical Digital Twin (BDT) Design Challenges



# BDT Design Challenge(s) that can be addressed by this resource

## **Sensitivity Analysis (SA) and VVUQ for BDT:**

- Quantify levels of uncertainty in models, parameters and data.
- Quantify role of machine learning in mechanistic models.
- Determine where/when data should be obtained to update models.
- Real-time computing and large-scale storage is continuing challenge.
- Surrogate and reduced-order models critical for real-time implementation – must permit accurate out-of-data predictions.
- Code verification generally necessary for all models.

# My resource

## **Sensitivity Analysis/Uncertainty Quantification:**

- Sensitivity analysis employed to determine subsets of influential parameters.
- Model parameters estimated via optimization, statistical inference, or data assimilation techniques.
- Experimental design to guide where to collect future data to best inform models.
- Code verification to establish accuracy; e.g., "Method of Manufactured Solutions".
- Uncertainty quantification to guide design, validation, and to assess risk.

# NASEM BDT Design Principles

- Sensitivity analysis provides a broad framework to determine parameters, data, and responses for specific individuals.
- Uncertainty analysis employed to quantify degree to which BDT is effective for considered individuals.
- Associated mathematical/statistical/numerical framework can be scaled based on available data and knowledge.
- Statistical mixed-effects and measure transport provide framework to construct virtual populations.
- Considered framework is inherently modular and can be adapted to accommodate new information, data, and model constructs as they become available.

# “New” math, stats, comp solutions

- Highly robust linear algebra and sampling-based sensitivity analysis techniques employed to determine noninfluential model parameters.
- Robust sampling algorithms employed to implement Bayesian inference of model parameters and experimental observation errors.
- Sample from parameter and error distributions to construct prediction intervals for Quantities of Interest; e.g., biomedical response for patient.
- Prediction intervals provide rigorous framework for validating BDTs.
- Prediction Intervals also employed to determine components of mechanistic models to be augmented via data-driven modeling.
- Design of experiments employed to determine where and when to collect additional data to improve predictions for BDTs.
- Reduced-order models employed for real-time implementation of BDTs.

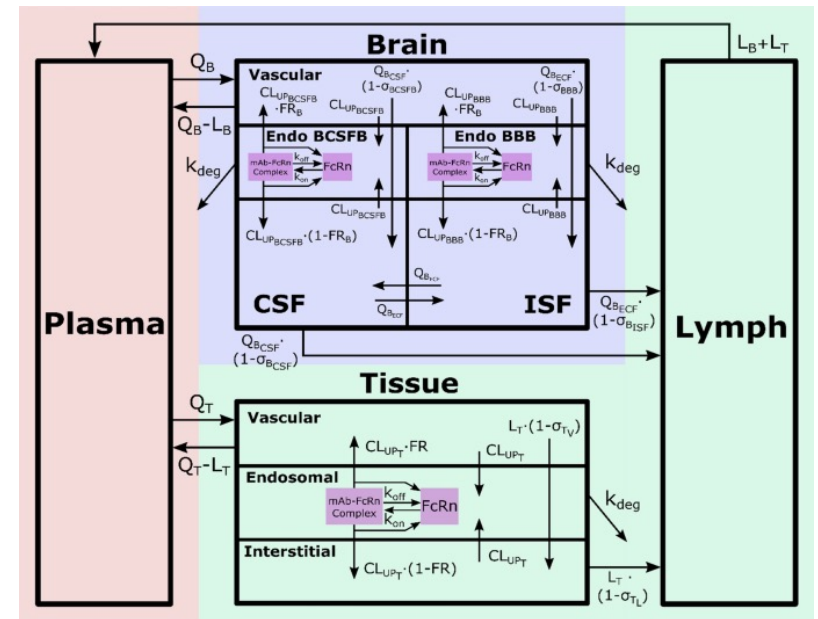
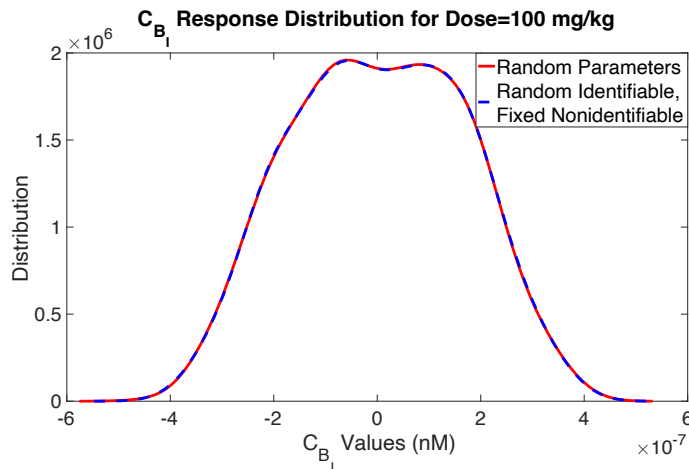


# “New” math, stats, comp solutions

**Example:** Minimal Physiologically-Based Pharmacokinetic (mPBPK) of brain for antibody therapeutics [Bloomingdale, Bakshi, Maass, et al., 2021]

**Note:** 16 ODE, 36 Parameters

**Step 1:** Employ sensitivity analysis to determine and verify 9 identifiable parameters

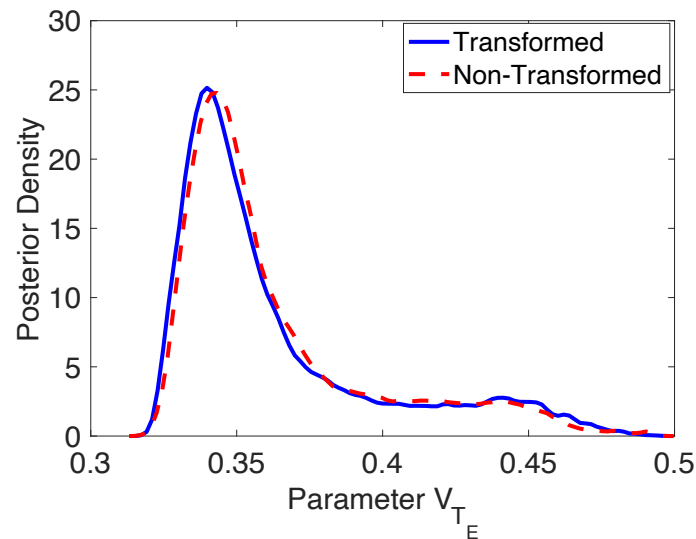




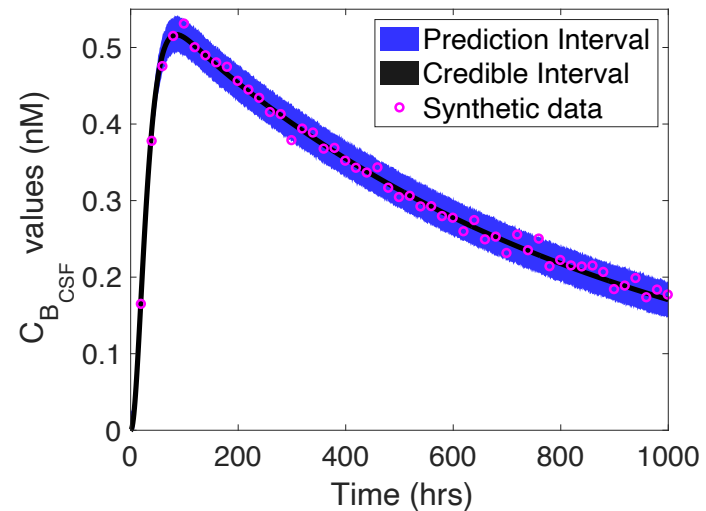
# “New” math, stats, comp solutions

## Step 2: Compute parameter and response distributions

### Parameter Distribution



### Prediction Interval for Response



## Current: Construct virtual population

# Integrating ethical design

- Use of digital twins may be used to reduce number of clinical trials to those which have proven safe and effective.
- Use of digital twins investigated to test novel and potential high-risk surgery techniques.
- Employ virtual populations to investigate safety, feasibility, and economic viability of considered procedures prior to clinical trials.

# References

- R.C. Smith, *Uncertainty Quantification: Theory, Implementation and Applications*, Second Edition, SIAM, Philadelphia, 2024.
- K. Dadashova, R.C. Smith and M.A. Haider, "Local Identifiability Analysis, Parameter Subset Selection and Verification for a Minimal Brain PBPK Model," *Bulletin of Mathematical Biology*, 2024.
- K.J. Pearce et al., "Modeling and parameter subset selection for fibrin polymerization kinetics with applications to wound healing," *Bulletin of Mathematical Biology*, 83(5), 47, 2021.
- M.T. Wentworth, R.C. Smith and B.J. Williams, "Bayesian model calibration and uncertainty quantification for an HIV model using adaptive Metropolis algorithms," *Journal of Inverse Problems in Science and Engineering*, 26(2), pp. 233-256, 2018.

# Questions

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